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PROFESSOR CAYLEY, President, in the Chair.

Rev. Samuel Jenkins Johnson, Upton Helions, near Crediton;
George Walter Roberts, Esq., Gomersal, near Leeds;
Alfred Herrtage, Esq., University College,

were balloted for and duly elected Fellows of the Society.

On a Pair of Differential Equations in the Lunar Theory.

By Prof. Cayley.

I consider the differential equations

$$\frac{d}{dt}\frac{d\varrho}{dt} - \varrho\left(\frac{dv}{dt}\right)^2 + \frac{1}{\varrho^2} = km^2\varrho\left\{\frac{1}{2} + \frac{3}{2}\cos(2v - 2mt)\right\},$$

$$\frac{d}{dt}\left(\varrho^2\frac{dv}{dt}\right) = jm^2\varrho^2\left\{-\frac{3}{2}\sin(2v - 2mt)\right\},$$

which when $j = k = 1$ give the following equations in the lunar theory ($D = t - mt$):

$$\frac{1}{\varrho} = 1 + \frac{1}{6}m^2 - \frac{179}{288}m^4 - \frac{97}{48}m^5 - \frac{757}{162}m^6 - \frac{4039}{432}m^7 - \frac{34751189}{1990656}m^8 - \frac{155067635}{4976640}m^9$$

$$+ \cos 2D \left[m^2 + \frac{19}{6} m^3 + \frac{131}{18} m^4 + \frac{383}{27} m^5 + \frac{510565}{20736} m^6 + \frac{23140781}{622080} m^7 + \frac{355021217}{9331200} m^8 + \frac{27888590059}{34992000} m^9 \right]$$

$$+ \cos 4D \left[\frac{7}{8} m^4 + \frac{2737}{480} m^5 + \frac{162869}{7200} m^6 + \frac{7554833}{108000} m^7 + \frac{2389416723}{12960000} m^8 + \frac{2335230125283}{5443200000} m^9 \right]$$

$$+ \cos 6D \left[\frac{219}{256} m^6 + \frac{151339}{17920} m^7 + \frac{29887443}{627200} m^8 + \frac{98978623957}{444528000} m^9 \right],$$

$$+ \cos 8D \left[\frac{2701}{3072} m^8 + \frac{700033633}{6021120} m^9 \right],$$

or as far as m^7,

$$\varrho = 1 - \frac{1}{6} m^2 + \frac{331}{288} m^4 + \frac{83}{16} m^5 + \frac{42775}{2592} m^6 + \frac{4787}{108} m^7$$

$$+ \cos 2D \left[- m^2 - \frac{19}{6} m^3 - \frac{125}{18} m^4 - \frac{709}{54} m^5 - \frac{485173}{20736} m^6 - \frac{24487949}{622080} m^7 \right]$$

$$+ \cos 4D \left[- \frac{3}{8} m^4 - \frac{1217}{480} m^5 - \frac{74069}{7200} m^6 - \frac{1749779}{54000} m^7 \right]$$

$$+ \cos 6D \left[- \frac{59}{256} m^6 - \frac{126193}{53760} m^7 \right],$$

($\frac{1}{\varrho}$ is given by M. Delaunay only as far as m^5, the additional terms of $\frac{1}{\varrho}$ and expression for ϱ were kindly communicated to me by Prof. Adams); and

$$v = t$$

$$+ \sin 2D \left(\frac{11}{8} m^2 + \frac{59}{12} m^3 + \frac{893}{72} m^4 + \frac{2855}{108} m^5 + \frac{8304449}{165888} m^6 + \frac{102859909}{1244160} m^7 + \frac{7596606727}{74649600} m^8 - \frac{805141816 1}{111974400} m^9 \right)$$

$$+ \sin 4D \left(\frac{201}{256} m^4 + \frac{649}{120} m^5 + \frac{647623}{28800} m^6 + \frac{31363361}{432000} m^7 + \frac{123030377303}{414720000} m^8 \right)$$

$$+ \sin 6\,D \left(\frac{3715}{6144} m^6 + \frac{664571}{107520} m^7 \right)$$

(Delaunay, t. xi. pp. 815, 836, 845.)

To integrate the original equations write

$$\varrho = 1 + \varrho_1 + \varrho_2 + \ldots$$
$$v = t + v_1 + v_2 + \ldots$$

where the suffixes indicate the degrees in the co-efficients k, j conjointly: the equations for ϱ_n, v_n take the form

$$\frac{d}{dt}\frac{d\varrho_n}{dt} - 3\varrho_n - 2\frac{dv_n}{dt} + V_n = Q_n,$$

$$\frac{d}{dt}\left(\frac{dv_n}{dt} + 2\varrho_n + U_n\right) = P_n,$$

where V_n, U_n, P_n, Q_n do not contain ϱ_n or v_n. From the second equation we have

$$\frac{dv_n}{dt} + 2\varrho_n + U_n = \Omega_n + \int P_n\,dt,$$

where Ω_n is a constant of integration, the integral $\int P_n\,dt$ containing only periodic terms; and then adding twice this to the first equation we have

$$\frac{d}{dt}\frac{d\varrho_n}{dt} + \varrho^n + V^n + 2U_n = 2\Omega_n + Q_n + 2\int P_n\,dt$$

which determines ϱ_n; and substituting its value in the other equation we have $\frac{dv_n}{dt}$, and thence v_n; the constant Ω_n is determined so that $\frac{dv_n}{dt}$ may contain no constant term. We have

$$V_1 = 0,$$

$$V_2 = -\left(\frac{dv_1}{dt}\right)^2 - \varrho 2_1\frac{dv_1}{dt} + 3\varrho_1^2,$$

$$V_3 = -2\frac{dv_1}{dt}\frac{dv_2}{dt} - 2\varrho_1\frac{dv_2}{dt} - \varrho_1\left(\frac{dv_1}{dt}\right)^2 - 2\varrho_2\frac{dv_2}{dt} + 6\varrho_1\varrho_2 - 4\varrho_1^3,$$

&c.

$$U_1 = 0,$$

$$U_2 = 2\varrho_1\frac{dv_1}{dt} + \varrho_1^2,$$

$$U_3 = 2\varrho_1\frac{dv_2}{dt} + (2\varrho_2 + \varrho_1^2)\frac{dv_1}{dt} + 2\varrho_1\varrho_2,$$

&c.

$$Q_1 = k m^2 \left(\tfrac{1}{2} + \tfrac{3}{2} \cos 2 D \right),$$

$$Q_2 = k m^2 \left\{ 3 v_1 \sin 2 D + \varrho_1 \left(\tfrac{1}{2} + \tfrac{3}{2} \cos 2 D \right) \right\},$$

$$Q_3 = k m^2 \left\{ - 3 v_2 \sin 2 D - 3 v_1^2 \cos 2 D + \varrho_1 v_1 \cdot 3 \sin 2 D + \varrho_2 \left(\tfrac{1}{2} + \tfrac{3}{2} \cos 2 D \right) \right\},$$

&c.

$$P_1 = j m^2 \left(- \tfrac{3}{2} \sin 2 D \right),$$

$$P_2 = j m^2 \left(- 3 v_1 \cos D - 3 \varrho_1 \sin 2 D \right)$$

$$P_3 = j m^2 \left\{ - 3 v_2 \cos 2 D + 3 v_1^2 \sin 2 D, - 6 \varrho_1 v_1 \cos 2 D + (2 \varrho_2 + \varrho_1^2) \cdot - \tfrac{3}{2} \sin 2 D \right\},$$

&c.

In particular attending to the values of P_1, Q_1 the equations for ϱ_1, v_1 are in their original form

$$\frac{d}{dt} \frac{d \varrho_1}{dt} - 3 \varrho_1 + 2 \frac{d v_1}{dt} = k m^2 \left(\tfrac{1}{2} + \tfrac{3}{2} \cos 2 D \right),$$

$$\frac{d}{dt} \left(\frac{d v_1}{dt} + 2 \varrho_1 \right) = j m^2 \left(- \tfrac{3}{2} \sin 2 D \right),$$

whence in the transformed form they are

$$\frac{d v_1}{dt} + 2 \varrho_1 = \Omega_1 + \frac{3 j m^2}{4 (1 - m)} \cos 2 D,$$

and

$$\frac{d^2 \varrho_1}{dt^2} + \varrho_1 = 2 \Omega_1 + k m^2 \left(\tfrac{1}{2} + \tfrac{3}{2} \cos 2 D \right) + \frac{\frac{3}{2} j m^2}{1 - m} \cos 2 D.$$

Thus the constant term of ϱ_1 is $2 \Omega_1 + \frac{1}{2} k m^2$, giving in $\frac{d v_1}{dt}$ a constant term $- 3 \Omega_1 - k m^2$; this must vanish, or we have $\Omega_1 = - \frac{1}{3} k m^2$; and the equations thus become

$$\frac{d v_1}{dt} + 2 \varrho_1 = - \frac{1}{3} k m^2 + \frac{3 j m^2}{4 (1 - m)} \cos 2 D,$$

$$\frac{d^2 \varrho_1}{dt^2} + \varrho_1 = - \frac{1}{6} k m^2 + \left(\frac{3}{2} k m^2 + \frac{\frac{3}{2} j m^2}{1 - m} \right) \cos 2 D,$$

and then completing the integration

$$\varrho_1 = - \frac{1}{6} k m^2 + \left\{ \frac{- \frac{3}{2} k m^2}{3 - 8 m + 4 m^2} + \frac{- \frac{3}{2} j m^2}{(1 - m)(3 - 8 m + 4 m^2)} \right\} \cos 2 D,$$

$$v_1 = \left\{ \frac{\frac{3}{2} k m^2}{(1 - m)(3 - 8 m + 4 m^2)} + \frac{\frac{3}{2} j m^2 (7 - 8 m + 4 m^2)}{(1 - m)^2 (3 - 8 m + 4 m^2)} \right\} \sin 2 D.$$

which are the accurate values of ϱ_1 and v_1.

Expanding as far as m^6 we have

$$\varrho_1 = k\left(-\tfrac{1}{6}m^2\right) + \cos 2\,\mathrm{D}\Big\{ k\left(-\tfrac{1}{2}m^2 - \tfrac{4}{3}m^3 - \tfrac{26}{9}m^4 - \tfrac{160}{27}m^5 - \tfrac{968}{81}m^6\right)$$

$$+ j\left(-\tfrac{1}{2}m^2 - \tfrac{11}{6}m^3 - \tfrac{85}{18}m^4 - \tfrac{575}{54}m^5 - \tfrac{3661}{162}m^6\right)\Big\}$$

which for $j = k$ is $\quad = k\left(-\ m^2 - \tfrac{19}{6}m^3 - \tfrac{137}{18}m^4 - \tfrac{895}{54}m^5 - \tfrac{5597}{162}m^6\right)$

and

$$v_1 = \sin 2\,\mathrm{D}\Big\{ k\left(\tfrac{1}{2}m^2 + \tfrac{11}{6}m^3 + \tfrac{85}{18}m^4 + \tfrac{575}{54}m^5 + \tfrac{3661}{162}m^6\right)$$

$$+ j\left(\tfrac{7}{8}m^2 + \tfrac{37}{12}m^3 + \tfrac{589}{72}m^4 + \tfrac{1037}{54}m^5 + \tfrac{27331}{648}m^6\right)\Big\}$$

which for $j = k$ is $\quad = k\left(\tfrac{11}{8}m^2 + \tfrac{59}{12}m^3 + \tfrac{929}{72}m^4 + \tfrac{896}{27}m^5 + \tfrac{41975}{648}m^6\right)$.

I have, not in general, but for the value $j = k$, calculated ϱ_2 and v_2 as far as m^6: I have not made the calculation for ϱ_3 and v_3, but their values may be deduced from the foregoing values of ϱ, v; the final expressions (when $j = k$) of ϱ, $= 1 + \varrho_1 + \varrho_2 + \varrho_3 + \ldots$ and v, $= t + v_1 + v_2 + v_3 \ldots$ are

$\varrho = 1$

$$+ k\left(-\tfrac{1}{6}m^2 \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\right)$$

$$+ k^2\left(\qquad\qquad\qquad\qquad \tfrac{331}{288}m^4 + \tfrac{83}{16}m^5 + \tfrac{5113}{288}m^6\right)$$

$$+ k^3\left(\qquad\qquad\qquad\qquad\qquad\qquad -\tfrac{1621}{1296}m^6\right)$$

$$+ \cos 2\,\mathrm{D}\Big\{ k\left(-\ m^2 - \tfrac{19}{6}m^3 - \tfrac{137}{18}m^4 - \tfrac{895}{54}m^5 - \tfrac{5597}{162}m^6\right)$$

$$+ k^2\left(\qquad\qquad\qquad\qquad \tfrac{2}{3}m^4 + \tfrac{31}{9}m^5 + \tfrac{329}{27}m^6\right)$$

$$+ k^3\left(\qquad\qquad\qquad\qquad\qquad\qquad -\tfrac{2381}{2304}m^6\right)\Big\}$$

$$+ \cos 4\,\mathrm{D}\Big\{ k^2\left(\qquad\qquad\qquad\qquad -\tfrac{3}{8}m^4 - \tfrac{1217}{480}m^5 - \tfrac{76589}{7200}m^6\right)$$

$$+ k^3\Big(\qquad\qquad + \frac{7}{20} m^6\Big)\Big\}$$

$$+ \cos 6\,D \Big\{ k^3\Big(\qquad\qquad - \frac{59}{256} m^6\Big)\Big\}.$$

and

$v = t$

$$+ \sin 2\,D \Big\{ k \Big(\frac{11}{8} m^2 + \frac{59}{12} m^3 + \frac{929}{72} m^4 + \frac{896}{27} m^5 + \frac{41975}{048} m^6\Big)$$

$$+ k^2\Big(\qquad - \frac{1}{2} m^4 - \frac{41}{12} m^5 - \frac{43}{3} m^6\Big)$$

$$+ k^3\Big(\qquad\qquad - \frac{783}{2048} m^6\Big)\Big\}$$

$$+ \sin 4\,D \Big\{ k^2\Big(\qquad \frac{201}{256} m^4 + \frac{649}{120} m^5 + \frac{665263}{28800} m^6\Big)$$

$$+ k^3\Big(\qquad\qquad - \frac{49}{80} m^6\Big)\Big\}$$

$$+ \sin 6\,D \Big\{ k^3\Big(\qquad\qquad + \frac{3715}{6144} m^6\Big)\Big\};$$

which for $k = 1$ agree with the foregoing formulæ (verifying them as far as m^5); the present formulæ exhibit the manner in which the expressions depend on the several powers of the disturbing force.

On the Variations of the Position of the Orbit in the Planetary Theory. By Prof. Cayley.

It has always appeared to me that in the Planetary Theory, more especially when the method of the variation of the elements is made use of, there is a difficulty as to the proper mode of dealing with the inclinations and longitudes of the nodes, hindering the ulterior development of the theory. Considering the case of two planets m, m', and referring their orbits to any fixed plane and fixed origin of longitudes therein, let θ, θ' be the longitudes of the nodes, ϕ, ϕ' the inclinations ($p = \tan\phi \sin\theta$, $q = \tan\phi \cos\theta$, &c., as usual); then the disturbing functions for m, m' respectively are developed, not explicitly in terms of ϕ, ϕ', θ, θ', but in terms of Φ, the mutual inclination of the two orbits, and of Θ, Θ' the longitudes in the two orbits respectively of the mutual node of the two orbits; Φ and Θ, Θ' being functions (and complicated

ones) of φ, φ', θ, θ'. Moreover, although in the general theory of the secular variations of the orbits of the planetary system, θ, ϕ, &c., are, as above, referred to one fixed plane (the ecliptic of a certain date), yet in the theory of each particular planet it is the practice, and obviously the convenient one, to refer for such planet the θ, ϕ to its own fixed plane (the orbit of the planet at a certain date), the effect of course being that ϕ, and consequently p, q, instead of being of the order of the inclinations to the ecliptic, are only of the order of the disturbing forces. It has occurred to me that the last-mentioned plan should be adhered to *throughout;* viz., that for each planet m, the position of its variable orbit should be determined by θ, the longitude of its node, and ϕ, the inclination in reference to the appropriate fixed plane (orbit of the planet at a certain date) and origin of longitude therein. The disturbing functions for the planet m and m' will of course depend not only on θ, θ', ϕ, ϕ', but on the quantities Φ, Θ, Θ' which determine the mutual positions of the two fixed planes of reference and origins of longitude therein, *these last being however absolute constants not affected by any variation of the elements;* so that as regards the variation of the elements the disturbing functions are in fact given as *explicit* functions of the variable elements θ, θ', ϕ, ϕ'; and where ϕ, ϕ' and therefore also p, q, p', q' are only of the order of the disturbing forces.

I proceed to work out this idea, for the present considering the development of the Disturbing Function only as far as the first powers of p, q &c. For comparison with the ordinary theory, observe that in this theory the disturbing function contains only the *second* powers of the p, q &c., made use of therein; these are in fact of a form such as $P + p$, $Q + q$, . . . where P, Q are absolute constants and p, q . . . are the p, q, . . . of the present theory; the ordinary theory gives therefore in the disturbing function a series of terms involving $(P + p)^2$, $(P + p)(Q + q)$, ... which I now take account of only as far as the first powers of p, q, . . . viz., they are in effect reduced to $P^2 + 2Pp$, $PQ + Pq + Qp$, &c. . . . The present theory is thus not now developed to the extent of giving the p, q . . . of the ordinary theory in the more complete form as the solutions of a system of simultaneous linear differential equations, but only to the extent of obtaining for these p, q . . . respectively the terms which are proportional to the time.

I commence with the following subsidiary problem. Consider a spherical triangle A B C (sides a, b, c, angles A, B, C, as usual), and taking the side c as constant, but the angles A and B as variable, let it be required to find the variations of C, a, b in terms of variations dA, dB and the variable elements C, a, b themselves. Although the geometrical proof would be more simple, I give the analytical one, as it may be useful.

We have

$$\cos C = -\cos A \cos B + \sin A \sin B \cos c,$$

and thence

$$-\sin C\,dC = (\sin A\cos B + \cos A\sin B\cos c)\,dA + (\sin B\cos A + \sin A\cos B\cos c)\,dB$$

$$= \frac{\sin B\sin c}{\tan b}\,dA + \frac{\sin A\sin c}{\tan a}\,dB,$$

that is

$$-\frac{\sin C}{\sin c}\,dC = \frac{\sin B\cos b}{\sin b}\,dA + \frac{\sin A\cos a}{\sin a}\,dB,$$

or finally,

$$-dC = \cos b\,dA + \cos a\,dB.$$

Next

$$\sin a = \sin c\,\frac{\sin A}{\sin C},$$

or differentiating

$$\cos a\,da = \frac{\sin c}{\sin^2 C}\,(\sin C\cos A\,dA - \cos C\sin A\,dC),$$

or substituting for dC its value,

$$= \frac{\sin c}{\sin^2 C}\left\{dA\,(\sin C\cos A + \cos C\sin A\cos b) + dB\,.\,\cos C\sin A\cos a\right\}$$

$$= \frac{\sin c}{\sin^2 C}\left\{dA\,\frac{\sin A\sin b\cos a}{\sin a} + dB\cos C\sin A\cos a\right\},$$

hat is,

$$da = \frac{1}{\sin C}\left\{dA\,\frac{\sin A}{\sin a}\sin b + dB\cos C\sin A\right\} \div \frac{\sin C}{\sin c},$$

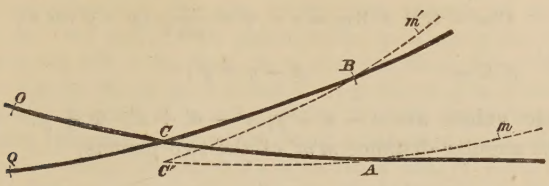
or on the right-hand writing $\frac{\sin A}{\sin a}$ instead of $\frac{\sin C}{\sin c}$ this is

$$da = \frac{1}{\sin C}\,(dA\sin b + dB\cos C\sin a).$$

And similarly,

$$db = \frac{1}{\sin C}\,(dB\sin a + dA\cos C\sin b).$$

Now let the continuous lines represent the orbits of m, m' at certain dates, O, Q the origins of longitude therein ; and the



dotted lines the variable orbits of the planets respectively.

Write

$$\begin{aligned} &OC = \Theta,\ CA = \theta,\ \angle CAC' = \varphi,\\ &QC = \Theta',\ CB = \theta',\ \angle CBC = \varphi',\\ &\angle C = \Phi. \end{aligned}$$

Then, answering to the rotation of the lemma, we have

$$a = \theta',\ b = \theta,\ C = \Phi, \qquad dA = \varphi,\ dB = -\varphi'$$
$$\text{or say} \qquad = \tan\varphi,\ = -\tan\varphi',$$

whence

$$C'B = a + da$$

$$= \theta' + \frac{1}{\sin\Phi}(\tan\varphi\sin\theta - \tan\varphi'\cos\Phi\sin\theta')$$

$$= \theta' + \frac{1}{\sin\Phi}(p - p'\cos\Phi),$$

$$C'A = b + db$$

$$= \theta + \frac{1}{\sin\Phi}(-\tan\theta'\sin\varphi' + \tan\varphi\cos\Phi\sin\theta)$$

$$= \theta - \frac{1}{\sin\Phi}(p' - p\cos\Phi),$$

$$\angle C' = C + dC = \Phi - \cos\theta\tan\varphi + \cos\theta'\tan\varphi'$$

$$= \Phi - q + q'.$$

Suppose v, v' are the longitudes of the planets in their two orbits respectively ; that is,

$$\begin{aligned} v &= OA + Am = \Theta + \theta + Am,\\ v' + QB + Bm' &= \Theta + \theta' + Bm', \end{aligned}$$

whence

$$C'm = C'A + Am = v - \Theta - \frac{1}{\sin\Phi}(p' - p\cos\Phi),$$

$$C'm' = C'B + Bm = v' - \Theta' + \frac{1}{\sin\Phi}(p - p'\cos\Phi),$$

$$\angle C' = \Phi - q + q';$$

say these values are $v - \Theta + x$, $v' - \Theta' + x'$, $\Phi + y$. Then if $\overline{H}$ is the angular distance mm' of the two planets,

$$\begin{aligned}\cos\overline{H} &= \cos(v - \Theta + x)\cos(v' - \Theta' + x') \\ &\quad + \sin(v - \Theta + x)(\sin v' - \Theta' + x').\cos(\Phi + y) \\ &= \cos(v - \Theta)\cos(v' - \Theta') + \sin(v - \Theta)\sin(v' - \Theta')\cos\Phi \\ &+ x\,[-\sin\overline{v - \Theta}\cos\overline{v' - \Theta'} + \cos\overline{v - \Theta}\sin\overline{v' - \Theta'}\cos\Phi] \\ &+ x'[-\cos\overline{v - \Theta}\sin\overline{v' - \Theta'} + \sin\overline{v - \Theta}\cos\overline{v' - \Theta'}\cos\Phi] \\ &+ y\,[-\sin\overline{v - \Theta}\sin\overline{v' - \Theta'}\sin\Phi]. \\ &= \cos H + \nabla \text{ suppose.}\end{aligned}$$

The disturbing function for the planet m disturbed by m' is

$$\Omega = m'\left\{\frac{1}{\sqrt{r^2 + r'^2 - 2rr'\cos\overline{H}}} - \frac{r\cos\overline{H}}{r'^2}\right\}.$$

($\Omega = -R$, if R is the disturbing function of the *Mécanique Céleste*); and the term hereof which involves ∇ is

$$\nabla\frac{d\Omega}{d.\cos\overline{H}}$$

where after the differentiation $\cos\overline{H}$ is replaced by $\cos H$

$$= m'\left\{\frac{rr'}{(r^2 + r'^2 - 2rr'\cos H)^{\frac{3}{2}}} + \frac{r}{r'^2}\right\}\nabla$$

viz., this a linear function of x, x', y, that is of p, q, p', q', with coefficients which of course involve the other variable elements and the time; but it will be remembered that Θ, Θ', Φ are not variable elements, but are absolute constants. The variations of p depend upon $\frac{d\Omega}{dq}$ and those of q on $\frac{d\Omega}{dp}$, and the quantities p, q, p', q' . . . disappear from these differential coefficients $\frac{d\Omega}{dq}$, $\frac{d\Omega}{dp}$; that is, disregarding periodic terms, and the variations of the elements, we obtain $\frac{dp}{dt}$, $\frac{dq}{dt}$ as absolute constants, or reckoning the time from the epoch belonging to the fixed orbit of m, we

have p, q as mere multiples of the time ($p = A\,t$, $q = B\,t$, where A and B are constants); agreeing with the statement preceding the investigation.

Observe that the p, q, as used above, have reference not only to the fixed orbit of m, but also to the node thereon of the fixed orbit of m': we may, if we please write, $p = \tan\phi \sin(\Theta + \theta)$, $q = \tan\phi \cos(\Theta + \theta)$, that is, $p = q \sin\Theta + p \cos\Theta$, $Q = q\cos\Theta - p\sin\Theta$, (or $p = P\cos\Theta - Q\sin\Theta$, $q = P\sin\Theta + Q\cos\Theta$) and in place of p, q introduce into the formulæ P and Q, which have reference only to the fixed orbit of m, and similarly writing $p' = \tan\phi' \sin(\Theta' + \theta')$, $q' = \tan\phi' \cos(\Theta + \theta')$, instead of p', q' introduce p', q' which have reference only to the fixed orbit of m'.

I remark that a table for the relative positions of the orbits of the eight Planets for the Epoch 1st January, 1850, is given in Leverrier's *Annales de l'Observ. de Paris*, t. ii. (1856), pp. 64–66.

On Uniformity in the Measurement of Position-Angles with the Telescope. By Capt. William Noble.

Inasmuch as no record exists of the *vivâ voce* discussions which take place at our meetings, I am induced to put into a more permanent form an oral suggestion which I made, on the spur of the moment, during a debate which arose on the night of January 12th last. My suggestion then was this: that one uniform system of measuring position-angles in the field of a telescope should be adopted. Every double-star observer in the world, without exception, measures the angles made with the

Fig. 1

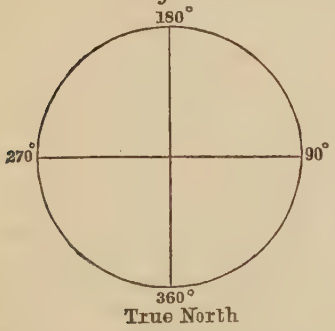
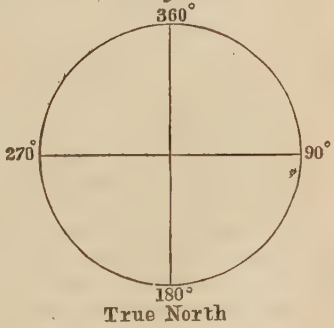


Fig. 2



meridian by a line joining the two components of the system, in the direction indicated in fig. 1; in which (assuming the ordinary Huyghenian or the Ramsden eye-piece to be employed) the bottom of the field is, of course, the true north. This, then, is, I may pretty confidently assert, the mode and direction in which any regular observer with the parallel-wire position-micrometer would naturally regard a circle, concentric with his visual telescopic field, to be divided. When, however, we come to study

the predictions of Occultations of Stars by the Moon given in the *Nautical Almanac*, we find the angles at which disappearance and reappearance are computed to take place no longer reckoned from the north round by east, south, and west, but having their initial point at the south, and being measured round by east, north, and west, as in fig. 2.

The reason for this I have utterly failed to discover. It seems to me to introduce a wholly needless complication, and one particularly embarrassing to the student and young astronomer; and I would therefore venture to submit that the mode of reckoning angles in the case of lunar occultations should be assimilated to that employed for measuring the position-angles of double stars. This need not, of course, prevent a supplementary calculation of the angle from the Moon's vertex, if thought necessary or desirable; but even this I would have measured round by the west, and not by the east, so as still to preserve the identity of the two systems.

Forest Lodge, Maresfield, March 2, 1872.

Note on an unsuspected Cause of Diffraction Phenomena in a Telescope. By Capt. William Noble.

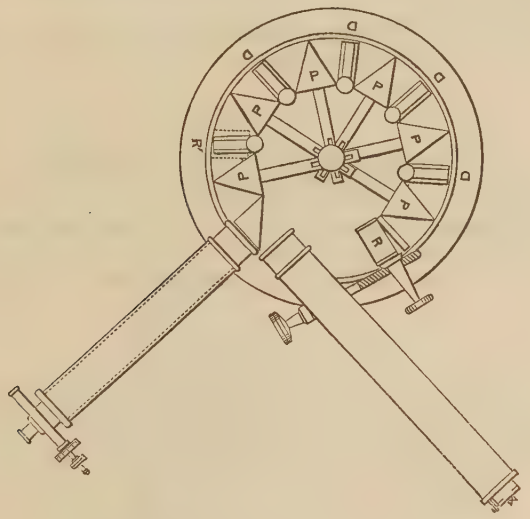
Some little time ago, in observing *Jupiter* and his satellites, I remarked certain emanations which appeared to have their origin in diffraction. I was very much puzzled to imagine in what way these phenomena could arise. My 4·2-inch Ross object-glass is simply perfect; my eyepieces were carefully cleaned, and, so far as I could see by removing them, and looking up the tube at the Moon or *Jupiter*, the tube itself was free from any obstruction. Since, however, the general definition of the instrument was sensibly unimpaired, I took no further action in the matter, and let things take their course until about the middle of last month. I was observing the Sun one morning at that period, when I removed the eyepiece for some reason, and happened to glance obliquely up the tube. To my astonishment I saw, brilliantly illuminated by the Sun, a perfect *grating* of excessively fine spider-webs, spun vertically across the interior of the telescope, somewhat within the focus of the object-glass. A light, in more senses than one, suddenly broke in upon me, and I very speedily removed the offending lines. I had the pleasure, that same evening, of viewing the Jovian system shorn of all optical appendages. I am too ignorant of the Arachnida to be able to guess at the species which produces a web of such extraordinary tenuity; but it certainly must be an extremely minute one, not only on account of the excessive fineness of the filaments which it spins, but also in order that it should have found its way inside of a tube so thoroughly and carefully closed as (I should think) to prevent the existence of interstice or aperture whatever whereby an entry might be effected.

Forest Lodge, Maresfield, March 2, 1872.

On an Universal Automatic Spectroscope.

By John Browning.

The diagram represents my automatic spectroscope, so made that any number of prisms from 1 to 6 may be used at pleasure; and as the ray is reversed, the power thus gained will vary from 2 to 12 prisms. This is achieved by so adjusting the right-angled prism that it may be slid into dovetails between each of the prisms, and the ray thus turned back at any part of its course. This can be done with as little trouble as is caused in changing the eye-piece of an ordinary microscope. None of the adjustments of the instrument need be interfered with, and the automatic motion is correct for any number of prisms. A simple contrivance is attached to the micrometer screw, so that it may be thrown out of gear, and the prisms set for any line at pleasure.



With sufficient care the adjustments of the prisms may be made so accurately that any line which falls on the spider-webs of the micrometer eye-piece will appear on the line when any number of the prisms are used. This forms the best method I have yet tried of accurately adjusting a spectroscope of high dispersive power, and enables me to obtain better definitions than I have hitherto done.

In the diagram P P &c. are the ordinary equilateral prisms. R is the reflecting prism shown at the end of the train; in which case, as the ray is reversed, a dispersive power equal to 12 prisms would be obtained. R′ shows the same prism in dotted lines, placed behind the first prism; thus giving the lowest dispersive

power with which the instrument could be used, equal to 2 prisms. D D D represents the dovetails in which the reflecting prism can be made to slide between any of the equilateral prisms.

In this instrument both the telescope and collimator remain stationary, so that neither the light nor the eye of the observer need be moved during a whole series of observations. By simply turning the micrometer-screw the train of prisms is moved, so that the whole of the spectrum passes before the eye of the observer, each ray, as it comes to the centre of the field of view, passing through the whole of the prisms at the minimum angle of deviation; thus ensuring the finest definition.

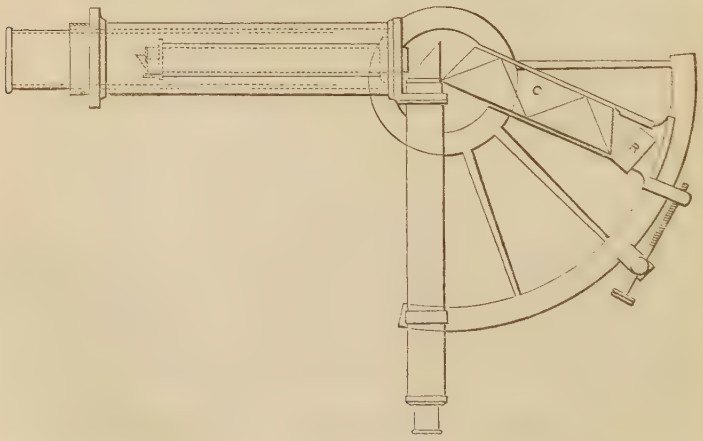
Such a spectroscope, fitted up as a tele-spectroscope, and used with a telescope above 6 in. in aperture, would answer equally well for observing the spectra of the stars, nebulæ, comets, planets, or the Sun, thus saving the multiplication of apparatus, and the great inconvenience of shifting the instruments when they were in adjustment.

While possessing these advantages, this instrument is at the same time more easy to use than any ordinary spectroscope.

On a Tele-Spectroscope for Solar Observations.

By John Browning.

The instrument made by me for Mr. Lockyer appears to answer its purpose very well, yet it leaves something to be desired on the score of lightness, and the manner of attaching



it to the eye-piece of the telescope makes it necessarily expensive. I have therefore sought to introduce an instrument which could be used with greater facility. This I have accomplished by

using a Rutherfurd's compound prism, as described in my paper, No. 7, vol. xxxi. of the *Monthly Notices*. The ray is reversed twice, so that the power of this prism is quadrupled. As the instrument is made as light as possible, it can be screwed into the eye-piece of a telescope in the same manner as an ordinary eye-piece, and it is sufficiently light to be used with a telescope of 4 in. in aperture.

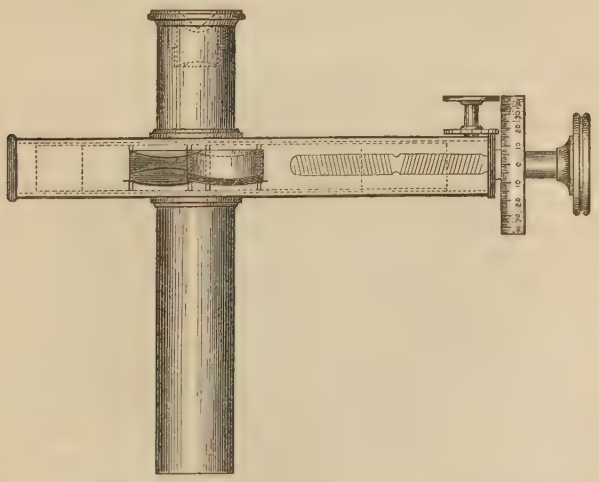
In the diagram C is the compound prism; R the reflecting prism by which the reversion of the ray is accomplished. A second prism of a similar kind is placed directly underneath this, in such a manner that it could not be shown in the diagram.

In using the instrument, it must be so arranged that the image of the Sun falls in focus on the slit.

The dispersive power of the instrument may be varied to suit the aperture of the telescope employed, so as to be equal to between 6 and 12 equilateral prisms of dense flint-glass.

On a Double-Image Micrometer. By John Browning.

As double-image micrometers can be used on a telescope without a clock-driving apparatus, and without the field of view being illuminated, they would doubtless be more employed if there were not important drawbacks to their general use.



Having recently considered the various contrivances used as double-image micrometers, I came to the conclusion that an instrument in many ways more advantageous might be made by simply dividing an ordinary Barlow lens, and causing the two

halves to move separately by means of a micrometer-screw. (See Diagram.) Having had this contrivance made, I find its performance even more satisfactory than I had anticipated. It is compact, inexpensive, and possesses the great advantage that it may be used with any eye-piece.

A Suggestion in the Use of Chronometers, with a view to its use in the approaching Transit of Venus. By David Gill, Jun., F.R.A.S.

Whilst equal increments of heat produce nearly equal effect of retardation on an uncompensated watch, the same increments of heat do not cause the weights of the ordinary chronometer compensation balance to approach the centre of the balance with equal increments.

Besides this, to produce perfect compensation the weights should approach to the centre with an increasing ratio to the temperature, and thus the result is to produce an irrationality of compensation, very analogous to the irrationality of spectra of crown or flint glass, producing secondary spectra in an object-glass.

It is evident from this that if such a balance is adjusted to produce perfect compensation at two very extreme temperatures, it will be over-compensated in mean temperatures.

Many circumstances have been made to correct this, notably by Mr. Hartnup, of Liverpool Observatory, by Messrs. Loseley, Kullberg, and others.

These have been attended with more or less success, but all require very great care in execution and much nicety in adjustment; so that it is still a question if the old form of balance, with certain precautions in its use, is not capable of giving the best results. What I would propose is this, that in the same box with the chronometer should be another chronometer, which we shall call the chronometric thermometer, provided either with an uncompensated brass balance, or a balance where the positions of brass and steel are reversed so as to exaggerate, instead of diminishing, the effects of temperature.

The *true* rate of the chronometer is then to be found for each rate of the chronometric thermometer relative to the chronometer.

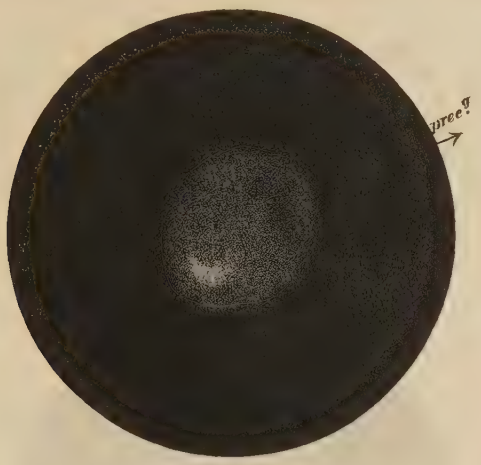
Both Greenwich and Liverpool now possess the most perfect means for ascertaining such rates, and could issue them in a tabulated form, with each instrument so tried.

If this were well carried out, and attention also paid to acceleration of mean rate, and allowance to be made for it, as pointed out by Mr. Hartnup, I am convinced that more perfect results than have been hitherto attained by chronometer transport could be arrived at.

Note on Encke's Comet. By the Rev. Henry Cooper Key.

(*Extract from Letter to Dr. Huggins.*)

I send some drawings I made of Encke's comet. The last one taken, on Dec. 3rd, differs slightly from your description as



Encke's Comet. Nov. 5, 1871.

I read it in some periodical. The train following the comet was

Encke's Comet. Nov. 8, 1871.

quite broad in my telescope, and could not be termed a "ray." You will observe two rays on the preceding side; these I have

drawn as you see, but I am *not perfectly certain* that the effect was not in my own eye and not a reality. I took every precaution to find out; and at the time (as well as now) felt pretty well convinced that it was no illusion. Four or five times I left the telescope, and, upon returning, there were the rays in exactly the same spot and direction. I feel pretty confident of their reality (they were extremely faint), but, as I say, am not *quite* certain, as I sometimes see dark lines in the field when first going to the telescope. The comet never seemed to me to lose its elliptical form from the first night I saw it, Oct. 20th. I detected a nucleus for the first time on Nov. 7th. The train I mentioned before was much fainter than the main body of the comet, and I was able to trace it to a distance of about 32′ from the nucleus. I saw nothing like the drawing of the comet made

Encke's Comet. Dec. 3, 1871.

at Greenwich, of which there is a woodcut in the November number of the *Notices*. In this woodcut the engraver, or some one, appears to have made a strange error as to the comet's direction of motion.*

The telescope I used was my 18-inch silvered glass Newtonian equatoreal; powers 92 and 142.

Stretton Rectory, Hereford,
Jan. 1st, 1872.

* Mr. Carpenter remarks that the dotted line through the comet was not intended as an indication of motion, but merely to show the inclination of the comet's axis to the equator.

Russian Preparations for the Observation of the Transit of Venus of 1874. By M. Otto Struve.

(*Extract from a Letter addressed to the Astronomer Royal*).

With regard to the preparation for the transit of *Venus*, I am glad to say that here in Russia all goes on in a regular way.

The inquiries on the meteorological conditions of the selected stations have given, on the whole, very satisfactory results, particularly for the stations on the coast of the Pacific Ocean, and in Eastern Siberia (about 85 per cent clear sky in December). There are only two stations, Taschkent and Astrabad, for which these conditions are not sufficiently satisfactory; therefore the observers designed for Taschkent will probably go to a place about 100 miles west of that town; and instead of Astrabad, we shall either take the island of Aschuradeh, in the Caspian Sea, or, if possible, cross the Elburz mountains and establish our observers at Schahrech in Persia, with nearly an absolute certainty of clear sky.

The total number of Russian stations will be twenty-four, but each of them will be provided with only *one* instrument for the transit-observation itself. We hope in this way to increase the meteorological chances. The instruments will be,—

1. Three 4-inch heliometers, ordered from Repsold.
2. Three photoheliographs, ordered from Dallmeyer.
3. Four 6-inch and four 4-inch equatoreals, ordered from Repsold; all provided with clockwork, filar micrometer, and spectral apparatus.
4. Ten 4-inch telescopes, designed merely for contact-observations.

Of course each station will be also provided with clocks, chronometers, and the instruments required for the exact determination of time. The principal instruments are already ordered, and most of them will be here in the course of the present or beginning of next year. For these instruments, also, the observers are in a great part already nominated. They will all come to Pulkowa for a certain time next year, to exercise themselves in the observations.

The geographical positions of the selected stations will not be determined by the transit-observers themselves; but all stations at which the transit has been successfully observed will afterwards be carefully determined by special expeditions of the General Staff of the Navy. For this purpose a principal line of telegraphic longitudes will be laid (probably next year) through all Siberia to Nicolajensk, with which line the other stations in that part of the empire can be easily joined, either by telegraphic or chronometric operations.

With regard to the photographic method, I can inform you that in two places—at Wilna, under the direction of Colonel

Smysloff, and at Bothkamp in Holstein by Dr. Vogel — they have perfectly succeeded in taking instantaneous photographs of the Sun with *dry* plates.

As far as I know, in Germany the preparations have not much advanced since last spring. The estimates have not yet been laid before the Reichtag, but it is expected that this will be done early in the spring. In the meanwhile Dr. Winnecke has been engaged with practical experiments concerning the heliometric method. He seems to be quite satisfied with the results. Also M. Paschen and Professor Seidel have been working on the photographic method, and Professor Zöllner on the application of the spectroscope for the observation of outer contacts. On these points I am not yet enabled to communicate positive results.

Pulkowa, February 26, 1872.

Eclipse of December last. Australian Expedition. By H. O. Russell, Esq., Government Observer at Sydney.

You are, no doubt, aware that the Royal Society of Victoria made arrangements to observe the recent total eclipse of the Sun. As the Government of New South Wales liberally contributed 300*l.* towards the expense of the expedition, I, with an observing party of five persons, joined the expedition to represent New South Wales. As no vessel offered in Victoria at any sum within the means of the promoters of the expedition, the *Governor Blackall* steamer was fitted out in Sydney for the expedition, and started, on the 27th of November, for Cape Sidmouth. Circumstances induced the observers to select No. 6 Woody Island in preference to the mainland, and upon that part of it (a mere sandbank), which is dry at high water, the two observatories (Victoria and New South Wales) were erected.

As all the instruments sent out by the Royal Society were used by the Victorian party, I was obliged to take such instruments as I could command in Sydney. These were a $7\frac{1}{4}$-inch refractor, by Merz and Son, mounted equatoreally, and driven by the clock; the eye-piece was removed, and a camera-box put in its place for the purpose of taking photographs during totality, a camera lens of 3-inch aperture and 30 inches focus, mounted in a very light camera, weighing when complete only 6 lbs., was attached to the tube of the refractor, and arrangements were made to uncover the telescope and camera by the same motion of the operator's hand. Two small telescopes, $2\frac{3}{8}$ and $2\frac{1}{7}$ inches apertures, mounted equatoreally, and driven by clockwork: in each of these position lines at right angles, were placed to facilitate making the drawings. Also, a $3\frac{1}{2}$-inch telescope, mounted equatoreally, and moved by hand. Lastly, tents and a complete pho-

tographic dark room, with 20 baths, and other things in proportion, to take 20 pictures during totality.

On the day they were landed all my instruments were put up and in part adjusted; during the next day adjustments were completed: and on the third day photographic practice began, and some good pictures were obtained. Experiment also convinced me that we might keep a plate out of the bath at a temperature of 80° for ten minutes, expose it and keep it ten minutes more, before developing, without any serious injury.

On Monday morning we had a full rehearsal and practice all day. All this time, though there had been a continued haziness in the atmosphere, we hoped that we might yet have it clear enough to see something. Monday evening, however, brought heavy clouds and a very severe thunder-storm, during which the lightning passed down the iron stays of the ship five times; and exploded between the stay and the hull, fortunately without damage. Heavy rain followed and continued at intervals all night and the next day; every preparation was, however, made, and nine plates prepared for exposure, even though it rained heavily, and no hope was left but in a possible rift in the clouds. One minute before totality I saw through dense clouds the line crescent of the Sun, but during totality nothing could be seen but a faint light in the clouds in the direction of the Sun; two plates were put in, and exposed 40 seconds, but not a trace of anything was obtained. During totality it was not dark: a peculiar blue grey, extending over clouds and landscape, gave yet abundance of light to read ordinary print. First and last contact were not seen: indeed, except at $1^h\ 40^m$ P.M., and one minute before totality, I saw nothing of the outline of the Sun.

During all the eclipse our sky was entirely overcast, and yet fifteen miles north of us the crew of a small schooner saw the eclipse, their attention being drawn to it by the darkness. It was quite evident from their evidence that there were light clouds over the Sun during totality; and although they state that there was a ring of light at the spot where they had seen the Sun, their description was very unsatisfactory and vague.

The two small telescopes driven by clockwork were taken in accordance with the instructions; larger ones could have been obtained if I had been able to get spectroscopes to use with them, but I could not.

Sydney Observatory, 29th December, 1871.

Note on the Nebula surrounding Eta Argûs. By H. O. Russell, Government Astronomer at the Sydney Observatory.

In Mr. Lassell's "Remarks on the supposed Change in the Great Nebula near *η Argûs*" there is one passage near the foot of p. 253 about which I have a few words of explanation to give.

The passage runs, "It is much to be regretted that Mr. Russell did not accompany his letter by a copy, more or less elaborate, of the drawing and stars he has so carefully laid down," &c.

On the 24th March, 1871, I sent to Sir John Herschel *four copies* of my drawing of the nebula in question, together with measures of the stars and notes of observation; also a statement of the dimensions and quality of the Sydney refractor. *On the 13th of May following* I wrote another letter to Sir John Herschel, enclosing a copy of the report which has got into Mr. Lassell's hands: that letter refers to information previously sent, and I can only regret that the drawings of the nebula there referred to were not put into Mr. Lassell's hands with the newspaper report.

December 29, 1871.

Measures of Binary Star ξ Ursæ Majoris. By George Knott, Esq.

I have obtained during the past two months the following three sets of measures of the interesting binary star *ξ Ursæ Majoris*, now near perihelion:—

P = 31°·60,	obs. 6,	w. 35;	D = 1″·092,	obs. 6.	w. 30;	Epoch 1872·028
29·69,	5,	29	1·111,	4,	24	1872·088
28·15,	5,	36	1·052,	4,	24	1872·138

The measures were taken with my 7½-inch Alvan Clark refractor and a parallel wire micrometer; magnifying powers 450 and 515.

Woodcroft Observatory, Cuckfield, Sussex, March 7, 1872.

Longitude of Teheran. By Lieut.-Col. J. T. Walker.

In September last an approximate value of the longitude of Teheran, which was required for the correction of the maps of Persia, was determined by Col. Walker, R.E., Superintendent of the Trigonometrical Survey of India, and Major St. John, R.E., of the Persian Telegraph Department, through the line of the Indo-

European Telegraph Company. The Directors of that Company had kindly granted the free use of the line for the purpose, and every assistance was most obligingly rendered by the Messrs. Siemens, who supervise all the arrangements for keeping the line in working order.

Signals were sent from London by Col. Walker, and from Teheran by Major St. John, the Greenwich times of the signals being ascertained from a clock in the Government Central Telegraph Office, which was governed by a clock in the Greenwich Observatory, while the Teheran times were determined by sextant observations taken on the spot by Major St. John, and his assistant, Capt. Pierson, R.E.

Considerable interest attaches to the operations from the circumstance that though the distance from London to Teheran, along the telegraph line, is nearly 4000 miles,* and it was necessary to employ automatic relays at five intermediate stations, the entire retardation of the electric current in either direction was found to average less than half a second. This shows that the line is in a very high state of efficiency, which is most creditable to the Messrs. Siemens, by whom all the working arrangements are carried out. Thus there is much reason to hope that, when the necessary instruments are available in India, exact and final determinations of the differences of longitude of the Greenwich and the Madras Observatories, and the stations on the arcs of parallel of the Indian Survey, may be obtained without any serious difficulty.

The value now determined for the longitude of Teheran is 51° 24′ 56″ east of Greenwich. It differs by less than half a minute, or say half a mile, from the value which had been previously deduced by Major St. John by combining a telegraphic determination of the difference between Teheran and Kurrachee, which was made by himself and his assistants, with the trigonometrical difference between Kurrachee and the Madras Observatory, which is furnished by the operations of the great Trigonometrical Survey of India, and assuming for the Madras Observatory the latest and most exact value of longitude, 80° 14′ 20″ east of Greenwich, which has been adopted by the Government astronomer at Madras, and is quoted in all the recent nautical almanacs. This close coincidence between two independent results, though possibly to some extent fortuitous, may be accepted as

*	Linden to Emden	390 miles.
	Emden to Berlin	330
	Berlin to Gitomis	850
	Gitomis to Kertch	800
	Kertch to Tiflis	700
	Tiflis to Teheran	800
	Total	3870 miles.

a sufficient proof that there can be no very material error in the adopted value for the Madras Observatory, and this is a matter of some importance, as all the most important determinations of longitude in India have invariably been referred differentially to that observatory.

Note on Colour as affected by Variation of Optical Power.
By Lt.-Col. A. Strange, F.R.S.

At one of the meetings of the Society last year, in a discussion on the changes of colour that had been observed at distant intervals in some of the planets, I stated my doubt whether the same observer at different epochs would be able to compare his impressions of colour satisfactorily. My belief is that of the three elements, form — size — colour, the last is that which is least permanently fixed, in most cases, in the sensorial memory.

A recent, perfectly unpremeditated, experiment bears upon the general question of estimating or distinguishing colour, though not upon the above particular branch of it. I was, within the last few days, at a theatre with two young ladies. They drew my attention to "the lady opposite in *pink*." Turning my glass to her, I replied, "You mean the lady in *yellow*." "No," replied both, "her dress is pink." Having ascertained that we all spoke of the same person, I begged my companions to use their glasses. On doing so they both at once admitted the colour to be yellow, as I had said. But they assured me that to their naked eye it was pink as before.

One of the young ladies, my own daughter, is considered to have a remarkably fine eye for colour, with the faculty of matching and remembering tints very strongly developed. The other, also a near relative, is likewise an excellent judge of colour, and a born artist. The dress about which the above doubt arose was not all coloured. It was white, with a great deal of the doubtfully-coloured trimming, and the tint (whatever it really may have been) was *very pale*. There was strong light upon it, and the distance was that of the whole greatest diameter of a small theatre.

Such are the facts, on which I do not propose to theorise. But they certainly point at least to one practical consideration—namely, the influence of optical power on colour impressions, and the necessity of great caution in pressing observations on the colour of heavenly bodies into the service of speculations regarding cosmical changes.

Since the above was written I have read Mr. Lassell's interesting paper on the planet *Jupiter* (*Monthly Notices* for January 1872), in which is the following passage,—"I may add further, that the variety of colour is most obvious with the higher powers, and when the atmosphere will not permit their use the tints are scarcely apparent."

Summary of Sun-spot Observations made at the Kew Observatory during 1871. Communicated by Messrs. De La Rue, Stewart, and Loewy.

Months.	Days of Observation.	Numbers given to the new Groups in the Kew Catalogue.	Number of New Groups.	Days without Sun-spots.
January	13	No. 1529 to No. 1547	19	0
February	16	1548 ,, 1576	29	0
March	26	1577 ,, 1619	43	0
April	21	1620 ,, 1647	28	0
May	25	1648 ,, 1677	30	0
June	21	1678 ,, 1698	21	0
July	21	1699 ,, 1719	21	0
August	23	1720 ,, 1740	21	0
September	19	1741 ,, 1752	12	0
October	14	1753 ,, 1765	13	0
November	13	1766 ,, 1786	21	0
December	7	1787 ,, 1799	13	0
Total	219	No. 1529 to No. 1799	271	0

A comparison of the numerical results with those published in the *Monthly Notices* of the Royal Astronomical Society for the year 1870, will prove that the latter year includes the maximum of Solar activity during the present period. The diminution is not only manifested in the numbers of the new groups, but also in their magnitude. Indeed, there is so marked an absence of really large and striking groups during the past year, that it more fully bears out our opinion, expressed on several previous occasions, that only by a continuous and well-devised plan of determining the absolute magnitudes of the Sun-spots from day to day, and from year to year, will light be ultimately thrown upon the true nature and relation of the forces, which regulate a series of phenomena of so well-established a cosmical interdependence that their discovery will undoubtedly mark a greater epoch in astronomical science than that of the universality of gravitation.

A striking feature in last year's observations seems to have been the occurrence of groups in comparatively high latitudes, especially in the Southern Solar hemisphere; a group observed between the 21st and 23rd of March had the almost unprecedented high latitude of $-43°$, while latterly, towards the end of the year, several groups in almost as high a latitude have repeatedly made their appearance. Such Polar groups appear to be the more short lived, the greater the distance from the well-defined equatorial region of common Solar activity; and there are indications that they are produced by sudden, but most violent, convulsions: the group of March, above alluded to, left long after its disappearance a ridge of faculous matter behind it, covering an area of at least twenty times the magnitude of the spot itself, and spreading out along a parallel of latitude more in the manner of a

cometary appendage than one of the usual faculæ which surround the central area of depression.

The Solar Prominences. By Father Secchi.

(*From the Comptes Rendus.* Abstract.)

"I have the honour of presenting to the Academy the result of all my observations made on the Prominences during the year 1871. I think it indispensable to present this summary, for it demonstrates that the conclusions, announced after the first observations, are confirmed by a sufficiently long period, comprising nine solar rotations. New facts also are made manifest, which bear importantly on the solar theory. The nine synodic rotations (approximate), and the number of days of effective observations, were distributed as follows :—

Rotation.	Days of observation.
I. April 23rd to May 21st . .	25
II. May 22nd to June 18th . .	24
III. June 19th to July 15th . .	26
IV. July 16th to August 12th . .	28
V. August 13th to September 9th .	25
VI. September 10th to October 7th .	18
VII. October 8th to November 9th .	14
VIII. November 5th to December 4th .	8
IX. December 5th to December 31st .	16

"The total number of prominences noted and figured amounts to 2667; the total number of days of complete observation is 184.

"The following table* contains the number of observed prominences arranged in bands of heliographic latitude ten degrees wide :—

	Number of Prominences.	
	Northern Hemisphere.	Southern Hemisphere.
90° to 80° . . .	118	139
80 ,, 70 . . .	142	180
70 ,, 60 . . .	73	87
60 ,, 50 . . .	87	61
50 ,, 40 . . .	125	156
40 ,, 30 . . .	182	183

* In the original paper the numbers are given in all the tables for each rotation. The tables so arranged would not be suited to these pages. Moreover, none of the results dwelt on by Secchi as depending on latitude, require the evidence of individual rotations. He considers the results for the whole Sun, with reference to the different rotations; and the necessary evidence for this purpose is here presented separately.

	Number of Prominences. Northern Hemisphere.	Southern Hemisphere.
30 „ 20 . . .	203	223
20 „ 10 . . .	195	228
10 „ 0 . . .	183	202
Sums .	1208 N.	1459 S.
Total	2667	
Mean number per day	14·49	

"We see that there are two principal maxima of frequency placed between 20 and 30 degrees of north latitude, and between 10 and 30 degrees of south latitude; two secondary maxima are found between 70 and 80 degrees in each hemisphere. The principal minima fall between 60 and 70 degrees north and 50 and 60 degrees south; one secondary minimum near the equator, between zero and 10 degrees north; and two other minima at the poles. In successive rotations, there is no trace of a progressive motion of the principal maximum towards the poles as had been suspected; if there is any small oscillation or displacement the phenomenon lasts but a short time; we shall see in future times whether this will be confirmed. The mean numbers of prominences per day of observation were as follows, for the rotations numbered I. to IX. above:—14·24; 16·12; 14·85; 15·78; 14·96; 14·61; 14·28; 13·75; 15·56; showing that the number of prominences reached a maximum between May and June, and a minimum between September and November. These variations correspond with the solar activity as deduced from the frequency of spots.

"But, to judge of the solar activity, the number of prominences is not enough; we must consider their height. For this purpose the following table has been prepared. It shows the mean height of all the observed prominences, this height being expressed in terms of an arbitrary unit, convenient for the observations, and corresponding to eight seconds (for convenience in drawing):—

	Mean Height of Prominences. Northern.	Southern.
90° to 80° . . .	5·08	5·45
80 „ 70 . . .	5·28	5·77
70 „ 60 . . .	4·27	4·73
60 „ 50 . . .	5·25	4·69
50 „ 40 . . .	5·81	5·93
40 „ 30 . . .	6·69	7·03
30 „ 20 . . .	6·32	6·92
20 „ 10 . . .	5·62	6·07
10 „ 0 . . .	5·90	6·00
Mean .	5·66	5·90
Absolute mean . . .	5·78	

"This table proves that the regions where the prominences are most numerous are those also where they are most lofty. The mean height diminishes from the second to the eighth rotation, or from May 22nd to December 4th, when it was reduced in the proportion of 4 to 7. (After this rotation it appeared to increase again.*) In the southern hemisphere the prominences are somewhat higher, as well as more numerous, than in the northern.

"The better to evaluate this interesting element, the following table has been prepared. It shows the number of prominences having a height exceeding 40 seconds, and the number exceeding 60 seconds in height :—

	Number of Prominences 40″ high and upwards.		Number of Prominences 60″ high and upwards.	
	Northern.	Southern.	Northern.	Southern.
90° to 80°	77	92	18	23
80 „ 70	110	148	33	43
70 „ 60	41	53	14	19
60 „ 50	52	33	26	7
50 „ 40	77	115	36	39
40 „ 30	134	140	65	67
30 „ 20	154	169	65	71
20 „ 10	132	175	40	69
10 „ 0	138	130	44	61
Sums	925	1039	343	408
Total	1964		751	
Mean number per day	10·67		3·22	

"It follows that the positions of the maxima and minima are nearly identical with those shown in the general table; and the secondary maximum, near the polar zone, remains very well marked. The total numbers observed during the rotations were as follows :—

	I.	II.	III.	IV.	V.	VI.	VII.	VIII.	IX.
Prominences above 40″	211	334	342	360	297	150	96	54	120
Prominences above 60″	100	142	164	132	107	34	25	10	37

"We see that after the fifth rotation there is a marked falling off in prominences of both orders.† This particular state of things corresponds visibly with the number of spots, which diminished greatly after September 10. The solar diameters also, as determined by the chronograph, showed greater regularity during this period of solar tranquillity.

* Or rather, perhaps, the diminution was too irregular to be properly indicated in periods of single rotation, as appears from the following numbers, giving the mean height for the nine rotations:—6·26, 7·38, 7·12, 6·25, 6·17, 5·04, 4·96, 4·05, 4·81.

† It will be remembered that the first order includes the second.

"In the following table the extent of the prominences in solar latitude is indicated, as well as the frequency of faculæ :—

			Mean width of Prominences.		Number of Faculæ.	
			Northern.	Southern.	Northern.	Southern.
90°	to	80°	5·02	5·12	61	61
80	„	70	5·82	5·56	75	94
70	„	60	4·49	5·05	83	106
60	„	50	5·10	5·51	95	100
50	„	40	5·68	5·37	150	142
40	„	30	5·99	6·01	223	231
30	„	20	6·12	5·83	295	317
20	„	10	6·06	6·03	280	315
10	„	0	5·81	5·66	170	192
		Mean	5·57	5·58	Sums 1432	1558
Absolute mean			5·575		Total 2990	

"We see that the regions which furnish the maximum numbers* are those also in which the prominences have their greatest extension. In successive rotations, diminished height corresponds to diminished width, as appears from the following mean widths for the nine successive rotations :—6·22, 5·65, 6·52, 5·48, 5·45, 4·75, 4·93, 4·87, 6·30.†

"The parallelism between height and width cannot be pursued further, for often a great extent of chromosphere is very high and very bright; but it does not reach higher than 24 seconds, the limit conventionally assigned to characterise a prominence. Be this as it may, however, we find that the prominence-masses which are highest are also those which have the greatest extent in longitude and latitude, though we meet sometimes with narrow and isolated prominences reaching to a great height.

"The classification of the positions of faculæ shows (1) that the regions richest in faculæ coincide with the regions already indicated where the prominences have their maximum height and width; (2) that the faculæ have a secondary minimum at the equator; and that beyond the maxima placed between 10 and 30 degrees, they decrease regularly up to the poles; but yet (3) that near the poles there are regions where rather bright granulations reach near to the border, and counting these we should find a secondary maximum of faculæ on the borders of the polar zones; but the limits are very difficult to observe, and have not always been taken into account.

* The text has been departed from here. Secchi writes:—"Les régions qui fournissent des maxima *et des minima* pour le nombre des protubérances sont aussi celles dans lesquelles les protubérances sont le plus étendues." The italicised words should obviously be omitted.

† Secchi does not sum up the number of faculæ in the different rotations, but their agreement with the prominences in this respect is close and significant, the faculæ being always most numerous when the prominences are so, and *vice versâ*.

"Among the 893 prominences observed between August 26th and December 31st, 471 were found to have a well-marked slope, resembling inclined plumes; of these, 370 were inclined in agreement with the law of movement of the solar atmosphere, from the equator towards the poles, and only 101 were sloped in the contrary direction; 40 were vertical, at the poles and equator. Comparing this result with that mentioned in former communications, we see that it cannot be accidental. I see that M. Spörer has arrived at the same result, later. I may remark here that during the epochs of greatest activity the law is more constant and decided."

Observations of the Planet (117) *Lomia, made at the Marseilles Observatory.* By MM. Stéphan and Borrelly.

Date of the Obser. 1871.	Hour of the Observation Marseilles Mean Time. d h m s	Apparent R.A. of (117) h m s	Log. f. Par.	Apparent Decl. (117) ° ′ ″	Log. f. Par.	Observer.	Comparison Star.
Sept. 12	10 47 12	23 49 6·25	+1·2477	+1 18 16·5	0·7759	B.	*a*
12	12 46 28	23 49 1·57	2·3885	1 18 11·1	0·7722	S.	*a*
13	9 45 41	23 48 14·47	1·4301	1 17 34·9	0·7772	B.	*a*
15	9 0 39	23 46 27·08	$\bar{1}$·5025	1 15 49·8	0·7783	B.	*f*
15	11 10 12	23 46 21·88	$\bar{1}$·0367	1 15 43·2	0·7753	S.	*b*
16	8 18 47	23 45 33·44	1·5582	1 14 53·6	0·7800	B.	*a*
18	10 11 1	23 43 36·39	1·2762	1 13 54·6	0·7763	S.	*b*
19	12 42 37	23 42 37·95	$\bar{2}$·9981	1 11 46·4	0·7762	S.	*c*
22	13 23 54	23 39 51·41	1·3027	1 8 38·7	0·7773	S.	*c*
29	21 36 32	23 33 41·13	2·8327	1 1 23·9	0·7775	S.	*d*
30	12 45 43	23 32 47·21	1·3027	1 0 18·2	0·7783	S.	*d*
Oct. 15	12 12 57	23 21 35·54	1·4107	+0 48 27·9	0·7805	S.	*e*

Adopted Mean Positions of the Comparison Stars, for 1871·0.

	Names of the Comparison Stars.	R.A. h m s	Decl. ° ′ ″	Magnitude.
a	25 Poissons 8303 B.A.C.	23 46 28·26	+1 22 26·7	6·5
b	869 Weisse, hora 23	23 43 31·95	+1 15 14·1	9·0
c	4779 Arg. Z. + 1°	23 43 22·10	+1 10 49·8	9·3
d	8243 B.A.C.	23 35 28·27	+1 4 20·0	5·0
e	5007 Arg. Z. + 0°	23 23 5·68	+0 54 56·8	9·4
f	874 Weisse, hora 23	23 43 45·43	+1 12 7·3	9·0

Nebulæ discovered and observed at the Marseilles Observatory.
By M. E. Stéphan.

Mean Positions for 1870·0.

Names of the Comparison Stars.		R.A. h m s	N.P.D. ° ′ ″	
281 Weisse (n.c) H. I.	9	1 12 3·05	74 21 25·9	Very very faint, very small, round, condensed at centre, but no bright point. (Scarcely perceptible.)
583 Weisse (n.c.) H. I.	8–9	1 29 18·33	48 20 28·8	Very very faint, very small, round, brighter at centre, but no bright point, properly so called.
1268 Weisse (n.c.) H. I.	8	1 54 53·40	58 33 30·8	Two neighbouring nebulæ. The two nebulæ are very very faint, very small; no bright points. The first is slightly more extended than the second, which is scarcely perceptible.
		1 55 46·82	58 33 7·3	
3971 Lalande	8½	1 58 29·25	77 22 17·8	Round, very small, very faint, more condensed at the centre, but no bright point, properly so called.
4713 Lalande	8½	2 23 28·11	59 49 52·1	Round, very small, not bright, bright points near the centre, seems resolvable.
465 Weisse (n.c.) H. II.	6–7	2 23 31·59	60 59 25·0	Very very small and faint, round, brighter at centre.
502 Weisse (n.c.) H. II	9	2 25 3·84	62 30 24·1	Extremely faint, small, gradually brighter towards the middle. Bright point.
655 Weisse (n.c.) H. II.	8	2 26 35·20	61 15 40·9	Moderately bright, of irregular form, lengthened in a direction inclined about 75° to the meridian; several bright points.
631 Rümker H. II.	6	2 27 18·08	67 9 31·1	Round, very very small, very faint, bright centre.
786 B.A.C.	6	2 27 19·20	59 3 6·4	Round, very very faint, very small, condensation at centre.
665 Rümker	9	2 30 17·32	49 2 13·8	Group of seven neighbouring nebulæ; all are extremely small and faint. No. 3 almost imperceptible; 1, 2, 5, and 6 have nearly the same brightness; 4 is rather diffuse, and somewhat more extended than 1, 2, 5, and 6, but fainter; 7 is less faint than the rest, a bright point in the middle.
,,		2 30 24·64	48 55 7·0	
,,		2 30 32·33	48 53 45·6	
,,		2 30 35·46	49 6 22·7	
,,		2 30 57·00	48 53 36·5	
,,		2 31 12·61	49 4 18·5	
677 Arg. Z. + 40°	7·1	2 33 55·63	49 3 20·3	
502 Arg. Z. + 34°	9·4	2 34 39·58	55 47 40·5	Extremely small and faint; a bright point.
1459 Weisse (n.c.) H. II.	9	3 4 9·68	51 10 24·7	Extremely faint, very small, round, no bright point, somewhat diffuse.

Mean Positions adopted of the Comparison Stars, for the Epoch 1870·0.

Names of the Comparison Stars.		R.A.	N.P.D.
		h m s	o ′ ″
281 Weisse (n.c.) H. I.	9	1 14 57·93	74 17 51·4
583 Weisse (n.c.) H. I.	8–9	1 27 38·10	48 16 14·6
1268 Weisse (n.c.) H. I.	8	1 54 12·99	58 30 37·0
3971 Lalande	8½	2 2 46 50	77 26 25·5
4713 Lalande	8½	2 27 53·72	55 52 50·1
465 Weisse (n.c.) H. II.	6·7	2 20 32·93	60 54 44·9
502 Weisse (n.c.) H. II.	9	2 22 0·27	62 32 22·4
655 Weisse (n.c.) H. II.	8	2 28 3·71	61 20 54·4
631 Rümker H. II.	6	2 21 49·24	67 6 46·9
786 B.A.C.*	6	2 24 11·23	58 59 27·6
665 Rümker H. II.	9	2 29 14·28	48 57 27·8
677 Arg. Z. + 40°	7·1	2 35 26·70	49 3 23·4
502 Arg. Z. + 34°	9·4	2 35 46·86	55 45 47·9
1459 Weisse (n.c.) H. II.	9	3 1 58·38	51 10 29·8

Erratum by Mr. Hunt.

The wood-engraver has made an important omission in the copy of the little map of Struve's stars, at page 91 of the *Monthly Notice* for January. He has omitted the star of Weisse's Bessel Hora xix, No. 745 of the 8th mag., at the centre of the trifolium where the three dotted lines meet. As the map is now drawn it represents simply a triangle, and requires the central star to answer to Sir W. Herschel's graphic description :—"A telescopic trifolium north following K, similar to that on the hand of *Aquarius*." Another omission was made in the map (not, however, important to the subject under discussion), namely, that of Weisse's Bessel, No. 692, 8·5 mag., and R.A., $19^h\ 26^m\ 11^s$, Dec.—10° 44′ 32″.

Chad Road, Edgbaston,
Feb. 5th, 1872.

* Another star must be intended. The place of 786 B.A.C. (15 *Trianguli*) for 1870·0 is in R.A. $2^h\ 17^m\ 52^s{\cdot}66$ and N.P.D. 55° 52′ 50″·88.